

Note on Laspeyres, Paasche and Fisher Indices¹

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The official CPI is based on a Laspeyres-type index, in which the weights are based in an historical period. Because consumers are believed to change their buying patterns away from higher inflation items over time, the Laspeyres index theoretically contains an inbuilt upward bias, the inverse is applicable to the Paasche index (both are fixed basket indices). The Fischer index is an approximation of a cost-of-living index (COLI). For purposes of analysis, Stats SA has calculated a Laspeyres, Paasche and a Fisher index.

The Paasche index

The Paasche index, P_P , is defined as:

$$P_P = \frac{\sum_{i=1}^n p_i^t q_i^t}{\sum_{i=1}^n p_i^0 q_i^t} \equiv \left\{ \sum_{i=1}^n (p_i^t / p_i^0)^{-1} s_i^t \right\}^{-1}$$

where s_i^t denotes the actual share of the expenditure on commodity i in period t , that is, $p_i^t q_i^t / \sum p_i^t q_i^t$.

The Paasche index is also called a “current weighted index”. It is a weighted *harmonic* average of the price relatives that uses the actual expenditure shares in the later period t as weights; whereas the Laspeyres index is the weighted arithmetic average that uses weights from a previous period. The harmonic mean is the mean of a set of numbers expressed as the reciprocal of the arithmetic mean of the reciprocals of the numbers. The harmonic mean of price ratios is defined as:

$$P_{HR}^{0:t} = \frac{1}{\frac{1}{n} \sum \frac{p_i^0}{p_i^t}}$$

The result is that the base period for both the Laspeyres and the Paasche is 100 when applying the arithmetic and harmonic mean respectively.

The Fisher index

The Fisher *price index*, P_F , is the *geometric* average of the Laspeyres and Paasche indices; and is defined as:

$$P_F \equiv \sqrt{P_L P_P}$$

The Fisher index gives a close approximation of the unknown COLI and will be between the Laspeyres and Paasche indices. The Laspeyres is normally the upper bound and the Paasche the lower bound indices.

The Laspeyres index: Adjustment of the 2008 weights to the current CPI methodology (2012 weights)

In order to compare the Laspeyres and Paasche indices and accurately calculate the Fisher index, the Laspeyres index (based on 2008 weights) should be updated to match with the most current weights (2012).

In 2008, the weights for food and non-alcoholic beverages, alcoholic beverages and tobacco, restaurants and personal care were adjusted for under-reporting. However, in 2012, additional adjustments were made to the weights that accounts for under-reporting as well as methodological requirements. The same categories as mentioned above were adjusted for under-reporting, and also adjusted was gambling. For methodological requirements, expenditure on restaurants and hotels were adjusted to include tourist expenditure, because Statistics South Africa used the domestic concept² when measuring inflation. Other methodological requirements include the use of the net weight for used vehicles and insurance³.

In order to make the two weights periods consistent, all these adjustments and methodological improvements were applied to the 2008 weights set. In addition, the weight set after adjustment was also price updated to the average of 2008, in a similar manner as that of the 2012 weight set.

The weights used to calculated the Paasche index are based on the 2012 weights set, as-is, and the Laspeyres index is based on the 2008 weights set (adjusted).

Table 1: Laspeyres and Paasche weights

COICOP Division	Laspeyres	Paasche
Food and non-alcohol beverages	20.59%	18.19%
Alcoholic beverages and tobacco	5.43%	5.37%
Clothing and footwear	3.74%	4.37%
Housing and utilities	23.40%	23.14%
Household contents and services	5.56%	4.93%
Health	1.38%	1.39%
Transport	14.80%	16.07%
Communication	2.60%	2.54%
Recreation and culture	4.65%	4.07%
Education	2.02%	2.66%
Restaurants and hotels	3.17%	3.33%
Miscellaneous goods and services	12.67%	13.94%

¹ Note that the Laspeyres as well as Paasche indices calculated are not exact indices but approximations of the two indices. The Paasche indices is the weighted harmonic mean of the elementary price indices and the Laspeyres indices, the weighted arithmetic mean of the indices.

² An index of inflation that covers all consumption expenditures that take place within the geographical boundaries of the country, whether made by residents or non-residents.

³ For further information refer to the Consumer Price Index Sources and Methods documents at www.statssa.gov.za

Comparison of the Laspeyres, Paasche and Fisher - theory

If the price and quantity changes (weighted by values) are *negatively* correlated, then the Laspeyres index exceeds the Paasche index. On the other hand, if the weighted price and quantity changes are *positively* correlated, then the Paasche index exceeds the Laspeyres index. Because consumers are usually price-takers, they typically react to price changes by substituting goods or services that have become *relatively* cheaper for those that have become *relatively* more expensive.; the *substitution effect*.

Substitution tends to cause a negative correlation between the price and quantity relatives, in which case the Laspeyres index is greater than the Paasche index, the gap between them tending to widen over time; in terms of index levels and not percentage change.

The normal result when calculating these indices are:

Laspeyres \geq Fisher \geq Paasche

Results of the Laspeyres, Paasche and Fisher indices⁴

For all three index types, the 12 divisions as well as the headline for all urban areas and total country were calculated. The three indices show the same result as explained in the theory section, the Laspeyres is the upper bound, the Paasche the lower bound and the Fisher is in the middle. The only exception is the housing and utilities division, which due to the high rates of price changes and weights in electricity, show the inverse.

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References:

1. *Consumer Price Index (CPI) ILO Manual*, International Labour Organization, <http://www.ilo.org/public/english/bureau/stat/guides/cpi/index.htm#manual>
2. *Glossary of Statistical Terms*, Organization for Economic Co-operation and Development (OECD), <http://stats.oecd.org/glossary/index.htm>

⁴ The data tables are available at www.statssa.gov.za (interactive data link)